EXAMPLE 2.1: FRM EXAM 2009—QUESTION 2-3

An analyst gathered the following information about the return distributions for two portfolios during the same time period:

Portfolio	Skewness	Kurtosis
A	−1.6	1.9
B	0.8	3.2

The analyst states that the distribution for Portfolio A is more peaked than a normal distribution and that the distribution for Portfolio B has a long tail on the left side of the distribution. Which of the following is correct?

a. The analyst's assessment is correct.
b. The analyst's assessment is correct for Portfolio A and incorrect for Portfolio B.
c. The analyst's assessment is not correct for Portfolio A but is correct for Portfolio B.
d. The analyst's assessment is incorrect for both portfolios.

EXAMPLE 2.2: FRM EXAM 2000—QUESTION 81

Which one of the following statements about the correlation coefficient is *false*?

a. It always ranges from −1 to +1.
b. A correlation coefficient of zero means that two random variables are independent.
c. It is a measure of linear relationship between two random variables.
d. It can be calculated by scaling the covariance between two random variables.

EXAMPLE 2.3: FRM EXAM 2007—QUESTION 93

The joint probability distribution of random variables X and Y is given by $f(x, y) = k \times x \times y$ for $x = 1, 2, 3$, $y = 1, 2, 3$, and k is a positive constant. What is the probability that $X + Y$ will exceed 5?

- **a.** 1/9
- **b.** 1/4
- **c.** 1/36
- **d.** Cannot be determined

EXAMPLE 2.4: FRM EXAM 2007—QUESTION 127

Suppose that A and B are random variables, each follows a standard normal distribution, and the covariance between A and B is 0.35. What is the variance of $(3A + 2B)$?

- **a.** 14.47
- **b.** 17.20
- **c.** 9.20
- **d.** 15.10

EXAMPLE 2.5: FRM EXAM 2002—QUESTION 70

Given that x and y are random variables and a, b, c, and d are constants, which one of the following definitions is *wrong*?

- **a.** $E(ax + by + c) = aE(x) + bE(y) + c$, if x and y are correlated.
- **b.** $V(ax + by + c) = V(ax + by) + c$, if x and y are correlated.
- **c.** $\text{Cov}(ax + by, cx + dy) = acV(x) + bdV(y) + (ad + bc)\text{Cov}(x, y)$, if x and y are correlated.
- **d.** $V(x - y) = V(x + y) = V(x) + V(y)$, if x and y are uncorrelated.

EXAMPLE 2.6: FRM EXAM 2002—QUESTION 119

The random variable X with density function $f(x) = 1/(b - a)$ for $a < x < b$, and 0 otherwise, is said to have a uniform distribution over (a, b). Calculate its mean.

a. $(a + b)/2$
b. $a - b/2$
c. $a + b/4$
d. $a - b/4$

EXAMPLE 2.7: FRM EXAM 2009—QUESTION 2-18

Assume that a random variable follows a normal distribution with a mean of 80 and a standard deviation of 24. What percentage of this distribution is *not* between 32 and 116?

a. 4.56%
b. 8.96%
c. 13.36%
d. 18.15%

EXAMPLE 2.8: FRM EXAM 2003—QUESTION 21

Which of the following statements about the normal distribution is *not* accurate?

a. Kurtosis equals 3.
b. Skewness equals 1.
c. The entire distribution can be characterized by two moments, mean and variance.
d. The normal density function has the following expression:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-\frac{1}{2\sigma^2}(x - \mu)^2]$$

EXAMPLE 2.9: FRM EXAM 2006—QUESTION 11

Which type of distribution produces the lowest probability for a variable to exceed a specified extreme value that is greater than the mean, assuming the distributions all have the same mean and variance?

a. A leptokurtic distribution with a kurtosis of 4
b. A leptokurtic distribution with a kurtosis of 8
c. A normal distribution
d. A platykurtic distribution

EXAMPLE 2.10: FRM EXAM 1999—QUESTION 5

Which of the following statements best characterizes the relationship between the normal and lognormal distributions?

a. The lognormal distribution is the logarithm of the normal distribution.
b. If the natural log of the random variable X is lognormally distributed, then X is normally distributed.
c. If X is lognormally distributed, then the natural log of X is normally distributed.
d. The two distributions have nothing to do with one another.

EXAMPLE 2.11: FRM EXAM 2007—QUESTION 21

The skew of a lognormal distribution is always

a. Positive
b. Negative
c. 0
d. 3

EXAMPLE 2.12: FRM EXAM 2002—QUESTION 125

Consider a stock with an initial price of \$100. Its price one year from now is given by $S = 100 \times \exp(r)$, where the rate of return r is normally distributed with a mean of 0.1 and a standard deviation of 0.2. With 95% confidence, after rounding, S will be between

a. \$67.57 and \$147.99
b. \$70.80 and \$149.20
c. \$74.68 and \$163.56
d. \$102.18 and \$119.53

EXAMPLE 2.13: FRM EXAM 2000—QUESTION 128

For a lognormal variable X, we know that $\ln(X)$ has a normal distribution with a mean of zero and a standard deviation of 0.5. What are the expected value and the variance of X?

a. 1.025 and 0.187
b. 1.126 and 0.217
c. 1.133 and 0.365
d. 1.203 and 0.399

EXAMPLE 2.14: FRM EXAM 2003—QUESTION 18

Which of the following statements is the most accurate about the relationship between a normal distribution and a Student's t distribution that have the same mean and standard deviation?

a. They have the same skewness and the same kurtosis.
b. The Student's t distribution has larger skewness and larger kurtosis.
c. The kurtosis of a Student's t distribution converges to that of the normal distribution as the number of degrees of freedom increases.
d. The normal distribution is a good approximation for the Student's t distribution when the number of degrees of freedom is small.

EXAMPLE 2.15: FRM EXAM 2006—QUESTION 84

On a multiple-choice exam with four choices for each of six questions, what is the probability that a student gets fewer than two questions correct simply by guessing?

a. 0.46%
b. 23.73%
c. 35.60%
d. 53.39%

EXAMPLE 2.16: FRM EXAM 2004—QUESTION 60

When can you use the normal distribution to approximate the Poisson distribution, assuming you have n independent trials, each with a probability of success of p?

a. When the mean of the Poisson distribution is very small
b. When the variance of the Poisson distribution is very small
c. When the number of observations is very large and the success rate is close to 1
d. When the number of observations is very large and the success rate is close to 0

2.7 ANSWERS TO CHAPTER EXAMPLES

Example 2.1: FRM Exam 2009—Question 2-3

b. Portfolio A has a longer left tail, due to negative skewness. In addition, it has less kurtosis (1.9) than for a normal distribution, which implies that it is more peaked.

Example 2.2: FRM Exam 2000—Question 81

b. Correlation is a measure of linear association. Independence implies zero correlation, but the reverse is not always true.

Example 2.3: FRM Exam 2007—Question 93

b. The function $x \times y$ is described in the following table. The sum of the entries is 36. The scaling factor k must be such that the total probability is 1. Therefore, we have $k = 1/36$. The table shows one instance where $X + Y > 5$, which is $x = 3, y = 3$. The probability is $p = 9/36 = 1/4$.

$x \times y$	$x = 1$	2	3
$y = 1$	1	2	3
2	2	4	6
3	3	6	9

Example 2.4: FRM Exam 2007—Question 127

b. The variance is $V(3A + 2B) = 3^2 V(A) + 2^2 V(B) + 2 \times 3\ 2\ \mathrm{Cov}(A, B) = 9\ 1 + 4\ 1 + 12 \times 0.35 = 17.2$.

Example 2.5: FRM Exam 2002—Question 70

b. Statement a. is correct, as it is a linear operation. Statement c. is correct, as in Equation (2.30). Statement d. is correct, as the covariance term is zero if the variables are uncorrelated. Statement b. is false, as adding a constant c to a variable cannot change the variance. The constant drops out because it is also in the expectation.

Example 2.6: FRM Exam 2002—Question 119

a. The mean is the center of the distribution, which is the average of a and b.

Example 2.7: FRM Exam 2009—Question 2-18

b. First convert the cutoff points of 32 and 116 into standard normal deviates. The first is $z_1 = (32 - 80)/24 = -48/24 = -2$, and the second is $z_2 = (116 - 80)/24 = 36/24 = 1.5$. From normal tables, $P(Z > +1.5) = N(-1.5) = 0.0668$ and $P(Z < -2.0) = N(-2.0) = 0.0228$. Summing gives 8.96%.

Example 2.8: FRM Exam 2003—Question 21

b. Skewness is 0, kurtosis 3, the entire distribution is described by μ and σ, and the p.d.f. is correct.

Example 2.9: FRM Exam 2006—Question 11

d. A platykurtic distribution has kurtosis less than 3, less than the normal p.d.f. Because all other answers have higher kurtosis, this produces the lowest extreme values.

Example 2.10: FRM Exam 1999—Question 5

c. X is said to be lognormally distributed if its logarithm $Y = \ln(X)$ is normally distributed.

Example 2.11: FRM Exam 2007—Question 21

a. A lognormal distribution is skewed to the right. Intuitively, if this represents the distribution of prices, prices can fall at most by 100% but can increase by more than that.

Example 2.12: FRM Exam 2002—Question 125

c. Note that this is a two-tailed confidence band, so that $\alpha = 1.96$. We find the extreme values from \$100exp$(\mu \pm \alpha\sigma)$. The lower limit is then $V_1 = \$100\exp(0.10 - 1.96 \times 0.2) = \$100\exp(-0.292) = \$74.68$. The upper limit is $V_2 = \$100\exp(0.10 + 1.96 \times 0.2) = \$100\exp(0.492) = \$163.56$.

Example 2.13: FRM Exam 2000—Question 128

c. Using Equation (2.46), we have $E[X] = \exp[\mu + 0.5\sigma^2] = \exp[0 + 0.5 * 0.5^2] = 1.1331$. Assuming there is no error in the answers listed for the variance, it is sufficient to find the correct answer for the expected value.

Example 2.14: FRM Exam 2003—Question 18

c. The two distributions have the same skewness of zero but the Student's t has higher kurtosis. As the number of degrees of freedom increases, the Student's t distribution converges to the normal distribution, so c. is the correct answer.

Example 2.15: FRM Exam 2006—Question 84

d. We use the density given by Equation (2.54). The number of trials is $n = 6$. The probability of guessing correctly just by chance is $p = 1/4 = 0.25$. The probability of zero lucky guesses is $\binom{6}{0}0.25^0 0.75^6 = 0.75^6 = 0.17798$. The probability of one lucky guess is $\binom{6}{1}0.25^1 0.75^5 = 6\ 0.25\ 0.75^5 = 0.35596$. The sum is 0.5339.

Note that the same analysis can be applied to the distribution of scores on an FRM examination with 100 questions. It would be virtually impossible to have a score of zero, assuming random guesses; this probability is $0.75^{100} = 3.2E - 13$. Also, the expected percentage score under random guesses is $p = 25\%$.

Example 2.16: FRM Exam 2004—Question 60

c. The normal approximation to the Poisson improves when the success rate, λ, is very high. Because this is also the mean and variance, answers a. and b. are wrong. In turn, the binomial density is well approximated by the Poisson density when $np = \lambda$ is large.